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# Magnetic and electric dipole constraints on extra dimensions and magnetic fluxes

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## Abstract

The propagation of charged particles and gauge fields in a compact extra dimension contributes to  $g - 2$  of the charged particles. In addition, a magnetic flux threading this extra dimension generates an electric dipole moment for these particles. We present constraints on the compactification size and on the possible magnetic flux imposed by the comparison of data and theory of the magnetic moment of the muon and from limits on the electric dipole moments of the muon, neutron and electron.

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## 1. Introduction

The possibility of Universal Extra Dimensions (UED) with size of the order of  $\text{TeV}^{-1}$  has been extensively studied for the past several years [1,2]; in UED models, all fields, not just gravity, have support in the extra dimensions. A size of these dimensions of the order of a  $\text{TeV}^{-1}$  allows for the possibility of observing their effects on low energy processes. The simplest enlargements of Minkowski space by one extra dimension, going back to Kaluza and to Klein [3], consists of replacing Minkowski space by  $M_4 \times S_1$ , where  $S_1$  is a circle of radius  $R$ . In this work we introduce a further modification by allowing a magnetic flux  $b/e$  to thread the circle  $S_1$ ; this will result in nontrivial periodicity for the phases of charged particle fields. In addition, such phases will make various interactions, specifically electromagnetism, P and T noninvariant allowing for the presence of electric dipole moments. It is this nonstandard source of CP violation that is one of the motivations for us to introduce a magnetic flux and to study its effects. Al-

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though, in the present work we restrict ourselves to the inclusion of only electromagnetic and charged fermion fields, we plan, in a future work, to extend this study to placing the full spectrum of the Standard Model in the five-dimensional geometry. In that case the charged gauge and Higgs fields acquire nontrivial periodicities resulting in masses for the lowest Kaluza–Klein mode gauge fields, even in the absence of the Higgs mechanism; the interplay between this mass generation and the one due to Higgs couplings is the second motivation for introducing the magnetic flux. In the present work we wish to obtain phenomenological relations between the compactification radius and flux and bounds on these and simplify the study, as mentioned above, to a simple spectrum of charged fermions and to photons.

Extra dimensions contribute to the anomalous magnetic moments, more specifically the anomalous gyromagnetic ratio,  $g - 2$ , of charged leptons. For the case of the muon this has been looked at previously [4,5] for various numbers of extra dimensions as well as to whether the lepton propagates in the bulk or is restricted to a brane. In the present work we allow for the fermions to propagate in the bulk, where the analysis performed in [4] shows that the effects of extra dimensions can be significant. Although our model is simpler than that of [4], in that we did not include contributions from  $W$  and  $Z$  exchanges, we find, in the case of no magnetic flux,  $b = 0$ , similar constraints on the compactification radius  $R$ . *We do find that limits on this radius do depend on the flux and for certain values of this flux the data tolerate larger compactification radii.*

In [5] contributions of the extra dimensions were evaluated to lowest order in a power series in  $(m_{\text{lepton}}R)^2$  for QED contributions and a series in  $(M_W R)^2$  for the exchange of other Standard Model particles. The latter is, by a factor  $(m_{\text{lepton}}/M_W)^2$ , smaller than the pure QED contribution (Eq. (48) of [5]). This suppression factor will also appear in terms involving the magnetic flux and that is why in this work we only considered QED fields propagating in the extra dimension. In the presence of the magnetic flux  $b$  the phenomenological analysis allows for larger values of  $R$  and thus we performed our calculation without relying on a power series expansion. In the  $b = 0$  case our limits on  $R$  agree with those of [5]. As mentioned earlier the effects of the magnetic flux on the propagation of all Standard Model fields will be studied in a separate work.

Constraints on the radius of the fifth dimension,  $R$ , for various values of the flux  $b$  are obtained by attributing the difference between theoretical and experimental values of muon magnetic dipole moment,

$$\delta_{(g-2)/2} = \frac{1}{2}(g-2)_{\text{exp}} - \frac{1}{2}(g-2)_{\text{th}}, \quad (1)$$

to the extra dimension. In turn, the flux  $b/e$  will be restricted by experimental limits on electric dipole moments (edm) of various elementary fermions. It will turn out that, for certain values of  $b/R$ , the corrections to the magnetic moment are very small allowing for large values of  $R$  and in turn large contributions to the edm's of some of the charged particles.

In restricting five-dimensional QED with a compact fifth dimension one encounters the problem of the fifth component of the gauge potential turning into an unwanted massless scalar particle. The standard method that prevents the appearance of such a state is to compactify the fifth dimension on the orbifold  $S_1/Z_2$  rather than on  $S_1$ . In addition to this orbifold compactification we shall also eliminate the unwanted massless fields by explicitly introducing a large mass for the fifth component. The values of  $\delta_{(g-2)/2}$  and of the edm's are significantly different for these two approaches. Both compactification formalisms are presented in Section 2. Generic results for  $\delta_{(g-2)/2}$  and for the edm are discussed in Section 3 while the numerical results and application to  $\delta_{(g-2)/2}$  of the muon and the edm's of the muon and neutron are discussed in

Section 4. In Section 5 we discuss a possible source of such fluxes and overall conclusions are presented in Section 6.

## 2. Five-dimensional QED with a magnetic flux

### 2.1. Compactification on a circle with explicit $A^5$ mass

On the space  $M_4 \times S_1$ , with  $M_4$  denoting ordinary Minkowski space-time and  $S_1$  a circle of radius  $R$ , the action for a charged four component fermion,  $\Psi(x^A)$ , and a gauge potential,  $A_B(x^A)$ , is

$$S = \int d^5x \left[ \bar{\Psi} (i \partial^A - e' A^A) \Gamma_A \Psi - m \bar{\Psi} \Psi - \frac{1}{4} F_{AB} F^{AB} + \frac{1}{2} A_A M^{AB} A_B \right]. \quad (2)$$

The upper case super and subscripts are the five-dimensional coordinates, with  $A = 0, 1, 2, 3, 5$ , the coordinates  $x^A = (x^\mu, y)$ , with  $0 \leq y \leq 2\pi R$  and  $F_{AB} = \partial_A A_B - \partial_B A_A$ . The five-dimensional Clifford algebra is spanned by  $\Gamma_A = (\gamma_\mu, i\gamma_5)$ . As discussed in the introduction, we allow for the possibility of giving  $A^5$  a mass by hand. We find it convenient to develop the formalism using a general mass matrix  $M^{AB}$ ; in the end four-dimensional Lorentz invariance will be reappear when only  $M^{5,5} \neq 0$ . The dimensionful  $e'$ , upon reduction to four dimensions, will be related to the ordinary electric charge  $e$  and to the radius  $R$  by

$$e' = \sqrt{2\pi R} e. \quad (3)$$

All neutral fields will be periodic under  $y \rightarrow y + 2\pi R$ ; the presence of a magnetic flux  $b$  threading the fifth dimension will change the charged field periodicities to

$$\Psi(x^\mu, y + 2\pi R) = e^{2i\pi b} \Psi(x^\mu, y); \quad \bar{\Psi}(x^\mu, y + 2\pi R) = e^{-2i\pi b} \bar{\Psi}(x^\mu, y); \quad (4)$$

and the phase  $b = e\Phi$  where  $\Phi$  is the flux threading  $S_1$ . The equations of motion, obtained by varying (2) are, as usual,

$$\begin{aligned} \partial_A F^{AB} + A_A M^{AB} &= e' \bar{\Psi} \Gamma^B \Psi, \\ \Gamma^B (i \partial_B - e' A_B) \Psi - m \Psi &= 0. \end{aligned} \quad (5)$$

Applying  $\partial_B$  to the first equation in (5) and using current conservation,  $\partial_B \bar{\Psi} \Gamma^B \Psi = 0$ , we obtain

$$M^{BA} \partial_B A_A = 0. \quad (6)$$

The case where only  $M^{5,5} \neq 0$  results in massless  $A_\mu$ 's and  $A_5$  independent of  $y$ .

We express all fields as a Fourier series in the  $y$  coordinate and impose the periodicity conditions of (4),

$$\begin{aligned} A^\nu(x^\mu, y) &= \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} A_n^\nu(x^\mu) e^{in(y/R)}, \\ A^5(x^\mu, y) &= \frac{1}{\sqrt{2\pi R}} A^5(x^\mu), \\ \Psi(x^\mu, y) &= \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} \psi_n(x^\mu) e^{i(n+b)(y/R)}. \end{aligned} \quad (7)$$

In terms of the Fourier coefficient fields,  $\psi_n(x^\mu)$ ,  $A_n^\nu(x^\mu)$ , and  $A^5(x^\mu)$  the action of (2) is

$$\begin{aligned} \mathcal{S} = \int d^4x \sum_{n=-\infty}^n & \left[ \bar{\psi}_n i \gamma^\mu \partial_\mu \psi_n - \bar{\psi}_n \left( m + i \frac{n+b}{R} \gamma_5 \right) \psi_n - \frac{1}{4} F_{n,\mu\nu} F_n^{\mu\nu} + \frac{n^2}{2R^2} A_{n,\mu} A_n^\mu \right] \\ & + \frac{1}{2} \left[ \partial_\mu A^5 \partial^\mu A^5 - M_{55}^2 (A^5)^2 \right] - e \sum_{n,m=-\infty}^{\infty} \bar{\psi}_n \gamma_\mu \psi_m A_{n-m}^\mu + -ie \sum_{n=-\infty}^{\infty} \bar{\psi}_n \gamma_5 \psi_n A^5. \end{aligned} \quad (8)$$

The fermion mass terms maybe expressed as  $m_n \bar{\psi}_n \bar{U}_n U_n \psi$ , with

$$\begin{aligned} m_n &= \sqrt{m^2 + \frac{(n+b)^2}{R^2}}, \\ U_n &= e^{i\beta_n \gamma_5}, \\ \cos 2\beta_n &= \frac{m}{m_n}; \quad \sin 2\beta_n = -\frac{n+b}{m_n R}. \end{aligned} \quad (9)$$

Using  $U_n \psi_n$  as fermion fields yields a conventional mass term  $m_n \bar{\psi}_n \psi_n$  at the price of complicating the interaction Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{int}} &= -e \sum_{m,n=-\infty}^{\infty} \bar{\psi}_n [\gamma_\mu \cos(\beta_n - \beta_m) + i \gamma_5 \gamma_\mu \sin(\beta_n - \beta_m)] \psi_m A_{m-n}^\mu \\ &\quad - e A^5 \sum_{n=-\infty}^{\infty} \bar{\psi}_n [\sin 2\beta_n + i \cos 2\beta_n \gamma_5] \psi_n. \end{aligned} \quad (10)$$

The transformations of all the fields under parity,  $\mathcal{P}$ , and time reversal,  $\mathcal{T}$ , are as usual, with the exception that  $n \rightarrow -n$ ,

$$\begin{aligned} \mathcal{P} \psi_n(\vec{x}, t) \mathcal{P}^\dagger &= \gamma_0 \psi_{-n}(-\vec{x}, t), \\ \mathcal{P} \bar{A}_n(\vec{x}, t) \mathcal{P}^\dagger &= -\bar{A}_{-n}(-\vec{x}, t), \\ \mathcal{P} A_n^0(\vec{x}, t) \mathcal{P}^\dagger &= A_{-n}^0(-\vec{x}, t), \\ \mathcal{P} A^5(\vec{x}, t) \mathcal{P}^\dagger &= -A^5(-\vec{x}, t), \end{aligned} \quad (11)$$

$$\begin{aligned} \mathcal{T} \psi_n(\vec{x}, t) \mathcal{T}^\dagger &= \gamma_1 \gamma_3 \psi_{-n}(\vec{x}, -t), \\ \mathcal{T} \bar{A}_n(\vec{x}, t) \mathcal{T}^\dagger &= -\bar{A}_{-n}(\vec{x}, -t), \\ \mathcal{T} A_n^0(\vec{x}, t) \mathcal{T}^\dagger &= A_{-n}^0(\vec{x}, -t), \\ \mathcal{T} A^5(\vec{x}, t) \mathcal{T}^\dagger &= -A^5(\vec{x}, -t). \end{aligned} \quad (12)$$

For  $b = 0$  the angle  $\beta_n = -\beta_{-n}$  and the action obtained from the above Lagrangian is invariant under both parity and time reversal. For  $b \neq 0$  the relation between  $\beta_n$  and  $\beta_{-n}$  no longer applies and both parity and time reversal are broken leading to the appearance of an electric dipole moment (edm). The product  $\mathcal{PT}$  is still conserved, precluding an induced anapole,  $\bar{\psi} \gamma_5 \gamma_\mu \psi \partial_\nu F^{\mu\nu}$ , [6] coupling.

$m_0$  is the mass of the lowest fermion in the KK tower, namely the one whose magnetic and electric moments we are interested in. By adjusting the input mass  $m$  in (9) we can set  $m_0$  equal to the physical mass, independent of the flux  $b$ . It is further convenient to set the  $m_0 = 1$  and

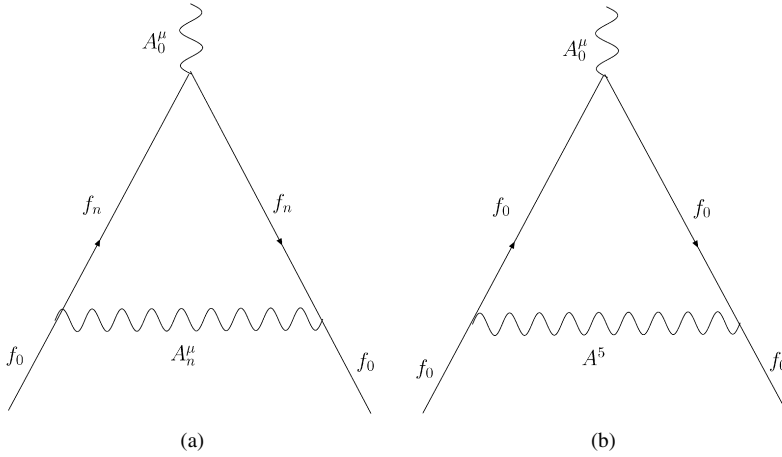


Fig. 1. Feynman diagrams for  $\delta_{(g-2)/2}$  and  $F_3$ ; (a) is for the exchange of KK photons and a fermion tower and (b) is for the exchange of a massive  $A^5$ .

express  $R$  in units of  $1/m_0$ ; with this convention we have

$$m_n = \sqrt{1 + \frac{2nb + n^2}{R^2}} \quad (13)$$

and the angles  $\beta_n$  of (9) satisfy

$$\cos 2\beta_n = \frac{\sqrt{1 - (b/R)^2}}{m_n}, \quad \sin 2\beta_n = -\frac{n+b}{m_n R}; \quad (14)$$

reality of  $\cos 2\beta_n$  requires  $|b| \leq R$ .

### 3. Contributions to the magnetic and electric dipole moments

#### 3.1. Massive $A^5$

Corrections to the gyromagnetic ratio of the fermion,  $\delta_{(g-2)/2}$  (1), and the value of its electric dipole moment,

$$\vec{d} = \frac{eF_3}{2m} \frac{\vec{\sigma}}{2}, \quad (15)$$

due to the extra dimension and magnetic flux are obtainable from the Feynman diagrams in Fig. 1. (For a different approach to extra-dimensional contributions to the anomalous magnetic moment see Ref. [5].) For a massive  $A^5$  we have

$$\delta_{(g-2)/2} = \frac{\alpha}{2\pi} \left[ \sum_{n=-\infty, n \neq 0}^{\infty} F_2(n, b, R; A_n^\mu) + F_2(b, R; A^5) \right],$$

$$F_3 = \frac{\alpha}{2\pi} \left[ \sum_{n=-\infty}^{\infty} F_3(n, b, R; A_n^\mu) + F_3(b, R; A^5) \right]; \quad (16)$$

the  $n = 0$  term in the summation for  $\delta_{(g-2)/2}$  is the usual first order correction,  $\alpha/2\pi$ , and thus excluded from the  $R$  and  $b$  dependent corrections. Although analytic expressions for all the terms appearing in (16) have been obtained, these are quite cumbersome. Rather, we shall present results as integrals over one Feynman parameter, which for subsequent analyses we evaluated numerically. For large  $n$  the terms in the summations behave as  $1/n$  making it appear divergent; however, this leading contribution cancels between  $n$  and  $-n$  resulting in a convergent series for both  $\delta_{(g-2)/2}$  and  $F_3$ .

For the  $F_2(n, b, R; A_n^\mu)$  we have

$$F_2(n, b, R; A_n^\mu) = \int_0^1 dz \frac{1-z}{z^2 - 2z(1 + nb/R^2) + m_n^2} \left\{ 4z(1 + bn/R^2) - 2z(1+z) - \frac{R^2}{n^2}(1-z)[z(1+m_n^2) - 2m_n^2 + (1+bn/R^2)(1-2z+m_n^2)] \right\},$$

$$F_2(b, R; A^5) = \int_0^1 dz \frac{(1-z)^2(z-1+2b^2/R^2)}{(1-z)^2 + zM_{55}^2}, \quad (17)$$

while for  $F_3$ ,

$$F_3(n, b, R; A_n^\mu) = \frac{n\sqrt{1-(b/R)^2}}{R} \int_0^1 dz \frac{(1-z)}{z^2 - 2z(1 + nb/R^2) + m_n^2} \left\{ 1 + 3z + \frac{2b}{n}(1-z) \right\},$$

$$F_3(b, R; A^5) = \frac{2b\sqrt{1-(b/R)^2}}{R} \int_0^1 dz \frac{(1-z)^2}{(1-z)^2 + zM_{55}^2}. \quad (18)$$

### 3.2. Orbifold compactification

When no mass is introduced for  $A^5$  this component of the gauge potential may be eliminated by a gauge transformation periodic in  $y$  with the exception of its  $n = 0$  Fourier coefficient. This results in an unwanted extra massless field. Compactifying the fifth dimension on an orbifold  $S_1/Z_2$ , which for practical purposes means that we expand all fields that appear at the  $n = 0$  level,  $\Psi$  and  $A^\mu$ , in even powers of  $n$ , while  $A^5$  in odd powers. Now a periodic gauge transformation eliminates  $A^5$  completely. The expressions in (16) are as before except the summation is only over even  $n$  and terms involving  $A^5$  are absent. The first term in an expansion in  $R^2$  agrees with Eq. (20) in [5].

## 4. Numerical results and discussion

### 4.1. $\delta_{(g-2)}$ limits

For various values of the flux, limitations on the compactification radius  $R$  are obtained from limits of the contribution of propagation in the extra dimensions to the anomalous magnetic moment (16). To this end we will use results presented by the E821 Collaboration [7] on  $(g-2)/2$  of the muon. Based on different theoretical evaluations of the hadronic vacuum polarization

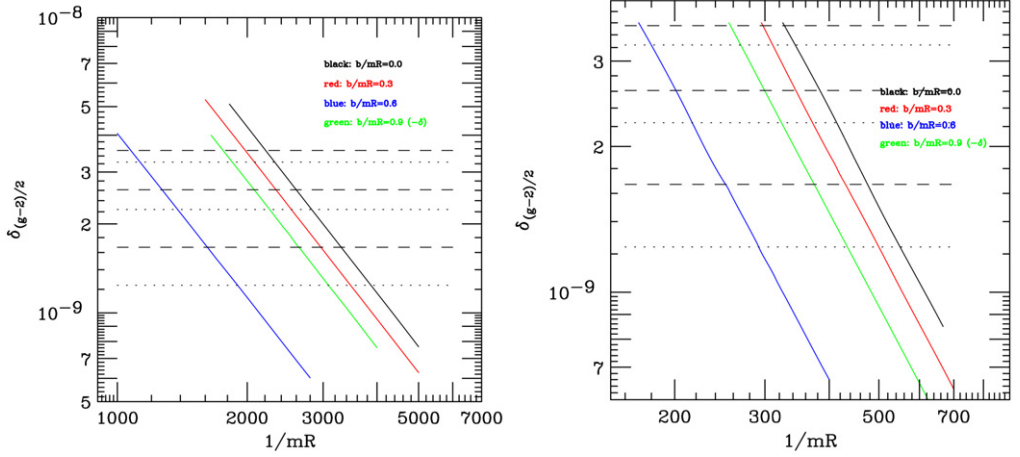


Fig. 2.  $\delta_{(g-2)/2}$  for various values of  $b/mR$  as a function of  $1/mR$ ; massive  $A^5$  (left) and orbifold compactification (right); note that for  $b/mR = 0.9$ ,  $\delta_{(g-2)/2}$  is negative. The horizontal lines indicate the central value and one standard deviation limits of the difference between Standard Model theory and experiment for  $(g-2)/2$  of the muon [7] for two different theoretical evaluations; dashed lines are for  $\delta_{(g-2)/2} = (2.61 \pm .94) \times 10^{-9}$  and the dotted ones are for  $\delta_{(g-2)/2} = (2.24 \pm 1.0) \times 10^{-9}$  (see text for details). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this paper.)

contribution to  $(g-2)/2$ , the results for  $\delta_{(g-2)/2}$  are  $(2.24 \pm 1.0) \times 10^{-9}$  and  $(2.61 \pm .94) \times 10^{-9}$ . The correction to the anomalous magnetic moment are presented in Fig. 2; for the case of a massive  $A^5$  we used the value of  $M_{55} = 1/R$ . (In this range of parameters the log-log plot is approximately linear. This is not true for lower values of  $1/mR$ , a region of no present interest.) For the case of a massive  $A^5$  these correspond to  $1/R = (2800^{+960}_{-550})m_\mu$  or  $1/R = (294^{+101}_{-58})$  GeV and  $1/R = (2600^{+600}_{-400})m_\mu$  or  $1/R = (273^{+63}_{-42})$  GeV. These limits are in the same range as those obtained in other analyses, e.g. Ref. [8]. With increasing values of the magnetic flux, the limits become progressively weaker. In the interval  $0.6 < b/mR < 0.9$ , and for the range of  $mR$  considered, the value of  $\delta_{(g-2)/2}$  changes sign.

We shall also note, Fig. 3, that for a given  $R$ ,  $F_3$  has a broad maximum for  $b/mR \sim (0.6-0.7)$ ; for upper limits on the edm's of various fermions we will set  $b/mR = 0.6$ . For this flux and for a massive  $A^5$  the acceptable values are  $1/R = 137^{+62}_{-16}$  GeV or  $1/R = 147^{+47}_{-20}$  GeV, while for orbifold compactification  $1/R = 21^{+6}_{-1}$  GeV and  $1/R = 22^{+8.5}_{-2}$  GeV. For the edm study we will use the combined range of  $(121 < 1/R < 197)$  GeV for massive  $A^5$  and  $(20 < 1/R < 30.5)$  GeV for orbifold compactification.

#### 4.2. Electric dipole moment limits

We now turn to results for electric dipole moments. These are presented in two ways: as a function of  $1/mR$  for various values of  $b/mR$ , Fig. 3, and as a function of  $b/mR$  for fixed  $R$ , Fig. 4. The log-log plots continue to be linear to larger values of  $1/mR$ . From Fig. 3 we note that  $F_3$  has, for the various parameters of this model, a broad maximum for  $b/R \sim 0.7$ .



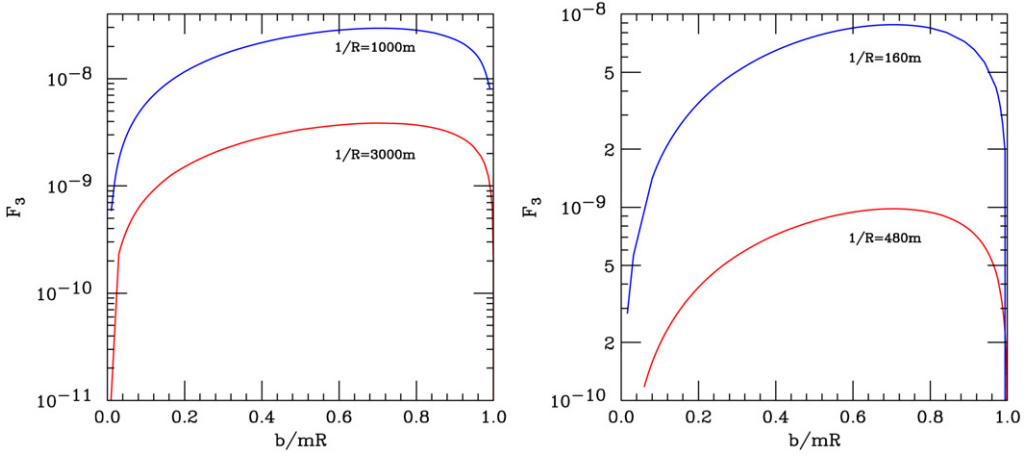


Fig. 3.  $F_3$  for various values of  $1/mR$  as a function of  $b/mR$ ; massive  $A^5$  (left) and orbifold compactification (right).

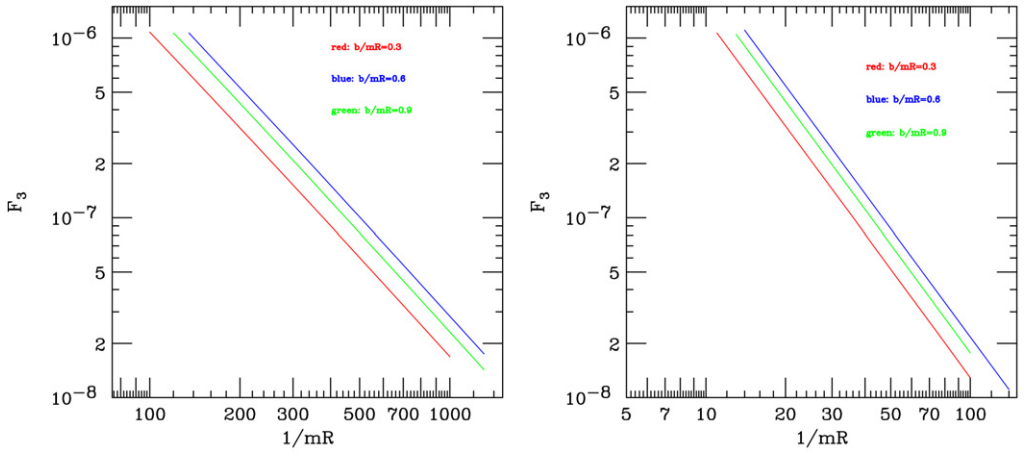


Fig. 4.  $F_3$  for various values of  $b/mR$  as a function of  $1/mR$ ; massive  $A^5$  (left) and orbifold compactification (right). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this paper.)

#### 4.2.1. Muon $edm$

Using the range of  $R$ , consistent with  $\delta_{(g-2)/2}$  obtained in Section 4.1, and setting  $b/m_\mu R = 0.6$  we obtain a maximum  $F_3$  which in turn provides an upper bound for the  $edm$  of the muon. For a massive  $A^5$  this bound is  $d_\mu \leq (2.6\text{--}30) \times 10^{-21}$  e-cm and  $d_\mu \leq (1.7\text{--}2.4) \times 10^{-19}$  e-cm for orbifold compactification. All these values are lower than the current experimental upper bound,  $d_\mu < 1.1 \times 10^{-18}$  [9].

#### 4.2.2. Neutron $edm$

A study, similar to that made for the muon, of limits imposed by the  $edm$  can be made for the neutron. In the nonrelativistic quark model the magnetic moments of the neutron and proton can be understood by assigning to quarks a Dirac magnetic moment and a mass of roughly one third that of the hadron [10]; this implies  $338 \leq 1/(m_{\text{quark}} R) \leq 563$  for massive  $A^5$  and

$63.5 \leq 1/(m_{\text{quark}} R) \leq 75$  for orbifold compactification. The bounds on the edm of the up-quark are  $d_{\text{up-quark}} \leq (0.7 - 2.0) \times 10^{-21}$  e-cm and  $d_{\text{up-quark}} \leq (7 - 8) \times 10^{-21}$  e-cm respectively. The experimental limit on the neutron's edm is  $d_n < 6.3 \times 10^{-26}$  e-cm [11]; in the quark model this is likewise the upper limit of the up quark's edm,  $d_{\text{up-quark}} < 6.3 \times 10^{-26}$  e-cm, smaller than the previous theoretical values. To reach the experimental limit while keeping  $R$  in the range determined by  $\delta_{(g-2)/2}$  of the muon, would require  $b/(m_{\text{quark}} R) \sim 10^{-5}$ .

#### 4.2.3. Electron edm

From  $d_e \leq 1.6 \times 10^{-27}$  e-cm [12] we obtain  $F_{3,e} \leq 5.0 \times 10^{-17}$ . Keeping  $R$  in the same range as in the previous analyzes, leads to  $2.4 \times 10^5 < 1/m_e R < 3.9 \times 10^5$  or  $3.9 \times 10^4 < 1/m_e R < 6 \times 10^4$  for the two schemes of handling  $A^5$ . For both cases this results in  $F_{3,e} \leq 10^{-12}$ . As with the limits imposed by the edm of the neutron, we need a further reduction of the magnetic flux by a factor of  $10^5$ .

### 5. Magnetic flux

For  $b = 0$  the residual discrepancy between theory and experiment on  $(g - 2)/2$  of the muon, Fig 2, results in an allowed range of compactification radius. Experimental limits on the electric dipole moments of various particles provide limits on the flux  $b$ . Muon edm results yield  $b \leq 1.5 \times 10^{-4}$  while those for the neutron and electron reduce this limit to  $b \leq 10^{-9}$ . If we assume that the magnetic induction,  $B$ , responsible for this phase is uniform over the area enclosed by the compactification radius  $R$  we obtain the relation  $b = eR^2 B/2$ . For  $1/R = 300$  GeV and, as above,  $b = 10^{-9}$  this results in  $B = 6 \times 10^{-5}$  GeV<sup>2</sup> or  $B = 1.3 \times 10^{12}$  T. Current loops carrying one ampere and spaced  $R$  apart would produce such a magnetic induction. Light particles moving along superconducting cosmic strings [13] generate currents of this magnitude.

### 6. Conclusion

The effects of charged lepton and photon fields propagating in a compact extra dimension, a circle of radius  $R$ , on the lepton's anomalous magnetic moment were considered. A magnetic field threading this extra dimension modifies the previous limits on  $R$  and induces a parity and time reversal violation permitting the appearance of an electric dipole moment. The extra, light, fifth component of the photon was eliminated in two ways: by the standard orbifold compactification or by giving it an *ad hoc* heavy mass. Ascribing the difference between the experimentally observed and a full Standard Model calculation of the magnetic moments of the electron and of the muon to the extra dimension places an upper bound on  $R$ . For no magnetic flux the best bound is obtained by comparing the muon magnetic moment; it is  $1/R \geq (273_{-42}^{+63})$  GeV. The presence of a magnetic flux weakens this limit down to  $1/R \geq (21_{-1}^{+6})$  GeV.

With the inclusion of a magnetic flux and for values of  $R$  discussed in the previous paragraph we obtain upper limits on the electric dipole moments,  $d_{\text{lepton}}$ , of the electron, muon and of the up and down quarks; the “experimental” limits on the latter are found from those for the neutron using a simple nonrelativistic quark model. For the muon the maximum theoretical value of  $d_\mu$  is below the current experimental bound. For the electron and for the quarks the maximum contribution due to the extra dimension exceeds the experimental limits, requiring a reduction in the value of  $b$  from the one that maximizes the electric dipole moments of these particles. The magnitude of such fields is consistent with that generated by light particles moving along superconducting cosmic strings at a distance of  $R$  from such a string.

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